Section 2.2 Derivatives of Products and Quotients (Minimum Homework: all odds)

We will learn two new rules to find derivatives in section 2.2.

The Product Rule – which is used when finding a derivative of problem with multiplication of two factors, both of which contain a variable.

The Quotient Rule – which is used when finding the derivative of a fraction that has a variable in the denominator.

Here are the rules and a short description of the symbols:

The Product Rule:

If $f \ and \ g$ are both differentiable functions, then:

$$\frac{d}{dx}\left(f(x) * g(x)\right) = f(x) * \frac{d}{dx} g(x) + g(x) * \frac{d}{dx} f(x)$$

Product Rule (derivative equals)

$$(first\ factor)(derivative\ of\ second\ factor) + second\ factor(derivative\ of\ first\ factor)$$

The Quotient Rule

If $f\ and\ g$ are both differentiable functions, then:

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x) * \frac{d}{dx}f(x) - f(x) * \frac{d}{dx}g(x)}{(g(x))^2}$$

Quotient Rule (derivative equals)

$$\frac{denominator(derivative\ of\ numerator) - numerator(derivative\ of\ denominator)}{(denominator)^2}$$

Example : Find the derivative using the Product rule.

$f(x) = (5x + 6)(x^2 - 3x)$

First: Determine the two factors. (parentheses are not required)

First factor: $5x + 6$ Second Factor: $x^2 - 3x$

Place them in the top row of a table:

First factor $5x + 6$	Second Factor $x^2 - 3x$

Second: find the derivative of each factor and put the derivative in the second row.

First factor $5x + 6$	Second Factor $x^2 - 3x$
Derivative 5	Derivative $2x - 3$

Third: Cross multiply top down and bottom up.

First factor $5x + 6$	Second Factor $x^2 - 3x$
Derivative 5	Derivative $2x - 3$
$cross\ multiply\ top\ down$ $(5x + 6)(2x - 3)$	$cross\ multiply\ bottom\ up$ $5(x^2 - 3x)$

Fourth: Add the expressions in the bottom row to find the derivative.

$f'(x) = (5x + 6)(2x - 3) + 5(x^2 - 3x)$

Fifth: Simplify

$f'(x) = 10x^2 - 15x + 12x - 18 + 5x^2 - 15x$

Answer: $f'(x) = 15x^2 - 18x - 18 \quad or \quad 3(5x^2 - 6x - 6)$

Example: Find the derivative using the Quotient rule.

$$f(x) = \frac{3x}{2x+5}$$

First: Create a table. Put the denominator in the top left position, the numerator in the top right position.

Denominator $2x + 5$	Numerator $3x$

Second: Find the derivative of each and put the result in the second row.

Denominator $2x + 5$	Numerator $3x$
Derivative 2	Derivative 3

Third: Cross multiply top down and bottom up.

Denominator $2x+5$	Numerator $3x$
Derivative 2	Derivative 3
$cross\ multiply\ top\ down$ $(2x+5)3 = 6x+15$	$cross\ multiply\ bottom\ up$ $2(3x) = 6x$

Fourth: create a fraction. Place the expressions in the numerator with a subtraction between. Place the square of the denominator in the denominator.

$$f'(x) = \frac{6x+15-6x}{(2x+5)^2}$$

Fifth: Simplify

$$f'(x) = \frac{15}{(2x+5)^2}$$

Answer: $f'(x) = \frac{15}{(2x+5)^2}$

Example: $f(x) = (5x + 6)(x^2 - 3x)$; x = 2

a) Find the slope of the tangent line to the graph of the function for the given value of x.

b) Find the equation of the tangent line to the graph of the function for the given value of x.

a) Slopes of tangent lines can be found by substituting $x = 2$ into the derivative.

$f'(x) = 15x^2 - 18x - 18 \ \ (from\ previous\ example)$

$m = f'(2) = 15(2)^2 - 18(2) - 18 = 6$

Answer $m = 6$

b) Need to find y-coordinate of the point.

$y = f(2) - (5(2) + 6)\big((2)^2 - 3(2)\big) = -32$

$point\ \ (2, -32)\ \ slope\ \ m = 6$

Equation of line

$y - (-32) = 6(x - 2)$

$y + 32 = 6x - 12$

Answer: $y = 6x - 44$

#1-12: Use the product rule to find the derivatives of the following.

1) $y = (2x + 3)(3x - 4)$

2) $y = (3x - 4)(5x - 8)$

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

Answer: $y' = 30x - 44$

3) $f(x) = (x - 2)(3x - 4)$

4) $y = (x - 5)(3x^2 + 7)$

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

$answer: \frac{dy}{dx} = 9x^2 - 30x + 7$

5) $f(x) = (x^2 + 3x + 2)(3x - 5)$

6) $f(x) = (3x^2 + 6x - 2)(4x + 1)$

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

$answer: \ f'(x) = 36x^2 + 54x - 2$

7) $g(t) = (2t - 1)(3t + 5)$

8) $g(t) = (3t^2 + 5t)(2t + 1)$

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

answer: $g'(t) = 18t^2 + 26t + 5$

9) $y = 3x^2(2x^2 + 6x - 4)$

10) $y = 4x^3(3x^2 + 7x - 5)$

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

$answer: \ y' = 4x^2(15x^2 + 28x - 15)$

11) $y = (3x^4)(5x^2 + 7)$

12) $y = (2x^5)(5x - 8)$

First factor	Second Factor
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

$answer: \; y' = 60x^5 - 80x^4 = \; 20x^4(3x - 4)$

#13-20: Use the quotient rule to find the derivative of the following.

13) $f(x) = \frac{6}{5x+1}$

14) $g(x) = \frac{4}{3x+11}$

Denominator	Numerator
Derivative	Derivative
cross multiply top down	*cross multiply bottom up*

Create a fraction. Place the expressions in the numerator with a subtraction between. Place the square of the denominator in the denominator

$answer: g'(x) = \frac{-12}{(3x+11)^2}$

15) $y = \frac{9x}{x-5}$

16) $y = \frac{12x}{5x-6}$

Denominator	Numerator
Derivative	Derivative
$cross\ multiply\ top\ down$	$cross\ multiply\ bottom\ up$

Create a fraction. Place the expressions in the numerator with a subtraction between. Place the square of the denominator in the denominator

$answer:\ y' = \frac{-72}{(5x-6)^2}$

17) $y = \frac{3t+1}{2t+5}$

18) $y = \frac{2t+3}{4t+5}$

Denominator	Numerator
Derivative Type equation here.	Derivative
$cross\ multiply\ top\ down$	$cross\ multiply\ bottom\ up$

Create a fraction. Place the expressions in the numerator with a subtraction between. Place the square of the denominator in the denominator

$answer: \frac{dy}{dx} = \frac{-2}{(4t+5)^2}$

19) $g(x) = \frac{x^2}{x-4}$

20) $g(x) = \frac{x^2}{x-2}$

Denominator	Numerator
Derivative 2	Derivative
$cross\ multiply\ top\ down$	$cross\ multiply\ bottom\ up$

Create a fraction. Place the expressions in the numerator with a subtraction between. Place the square of the denominator in the denominator

$answer\colon\ g'(x) = \frac{x(x-4)}{(x-2)^2}$

#21-26:

a) Find the slope of the tangent line to the graph of the function for the given value of x (or t).

b) Find the equation of the tangent line to the graph of the function for the given value of x (or t).

21) $y = (2x + 3)(3x - 4);\ x = 2$

22) $y = (3x - 4)(5x - 8)$; x = 3

(derivative computed in #1 / 2)

22a) $m = 46$

22b) $y = 46x - 103$

#21-26:

a) Find the slope of the tangent line to the graph of the function for the given value of x (or t).

b) Find the equation of the tangent line to the graph of the function for the given value of x (or t).

23) $g(t) = (2t - 1)(3t + 5)$;t = 4

24) $g(t) = (3t^2 + 5t)(2t + 1); t = -2$

(derivative computed in 7 / 8)

$answer\ \ 24a)\ m = 25 \quad 24b)\ y = 25t + 44$

#21-26:

a) Find the slope of the tangent line to the graph of the function for the given value of x (or t).

b) Find the equation of the tangent line to the graph of the function for the given value of x (or t).

25) $f(x) = \frac{6}{5x+1};\ x = 1$

26) $g(x) = \frac{4}{3x+11};\ x = -3$

(derivative computed in 13 / 14)

$answer\ \ 26a)\ m = -3 \quad 26b)\ y = -3x - 7$